Introduction to Transport Phenomena: Solutions Module 5

Solution 5.1

a) The shear stress is given by Newton's law of viscosity:

$$\tau_{yx} = -\mu \frac{dv_x}{dy}$$

For a linear velocity profile,

$$\boldsymbol{v_x = ay + b} \textbf{ (Eq.1)}$$

We know that, $v_x = 0$ at y$= 0$.
If we substitute this in Eq.1, we get$b = 0$
$v_x = 1.125 \frac{m}{s}$at y$= 0.075m$
So, if we substitute this in Eq.1, we get

$$1.125 = a \times 0.075$$

Or $a = 15$

$$v_x = 15y$$

Eq.2 represents the linear velocity profile
Thus,

$$\frac{dv_x}{dy} = 15$$

The shear stress at all the asked points is the same because $\frac{dv_x}{dy}$is a constant and does not vary with y.

$$\tau_{yx} = -0.048 \times 15 = -0.72N/m^2$$

b) The velocity profile is parabolic with maximum in y_1

$$\boldsymbol{v_x = a(y - 0.075)^2 + b \ (Eq.2)}$$

Here, we cannot take $v_x = a(y-0)^2 + b$ because the vertex of the parabola is not at the origin but is at y_1. If the vertex was at origin, we would get $y = a(v_x - 0)^2 + b$

We know that, $v_x = 0$ at y$= 0$.

$$0 = a(-y_1)^2 + b$$

$v_x = 1.125 \frac{m}{s}$at y$= 0.075m$

$$1.125 = +b$$

So, we get

$$0 = a(-0.075)^2 + 1.125$$
$$a = -200$$
$$v_x = -200(y - 0.075)^2 + 1.125$$

We need

$$\frac{dv_x}{dy} = \frac{d}{dy}(-200(y - 0.075)^2 + 1.125)$$

$$\frac{dv_x}{dy} = -400(y - 0.075)$$

So, we get

$$\tau_{yx} = -\mu \frac{dv_x}{d}$$

$$\tau_{yx}\big|_{y=0} = -0.048 \times -400 = -1.44\ Nm^{-2}$$

$$\tau_{yx}\big|_{y=0.025} = -0.048 \times -400 \times 0.05 = -0.96\ Nm^{-2}$$

$$\tau_{yx}\big|_{y=0.075} = 0\ Nm^{-2}$$

Note:

In order to determine the parabolic equation, it is possible to take the general equation of a parabola given by:

$$v_x = Ay^2 + By + C$$

We apply the following conditions:

$$y = 0, v_x = 0$$

$$y = y_1, v_x = v_1$$

$$y = y_1, \frac{dv_x}{dy} = 0$$

If we solve the above equation with 3 unknowns and 3 conditions to appl, we can solve for A, B and C.

We get the following equation:

$$v_x = -\frac{v_1}{y_1^2}y^2 + \frac{2v_1}{y_1}y$$

This is the same equation as Eq.2 and we get the same solution as shown above.

Solution 5.2

The plate in the center does not move ($v_0 = 0$) if the total shear stress applied to it is null, i.e if the stress from the fluid A compensates exactly the stress from fluid B.

$$\mu_A \frac{(v_1 - v_0)}{y_1} = \mu_B \frac{(v_2 - v_0)}{y_2}$$

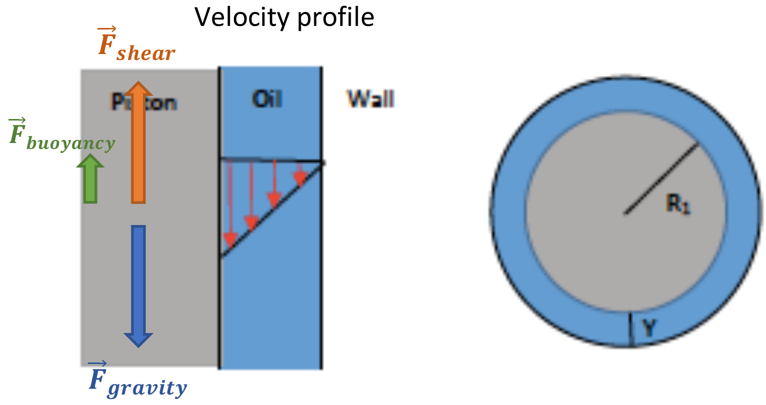
Therefore,

$$y_2 = \frac{\mu_B}{\mu_A} \times \frac{v_2}{v_1} \times y_1$$

$$y_2 = \frac{0.0008 \times 1 \times 0.05}{0.001 \times 2} = 0.02m$$

Fluid B needs to have a thickness of 2 cm for the center plate to be motionless.

Solution 5.3



The force balance on the car+piston gives:

$$\vec{F}_{tot} = \vec{F}_{gravity} + \vec{F}_{shear} + \vec{F}_{buoyancy}$$

The shear force is a reaction force to the movement from the fluid, thus it acts always in the opposite direction of the movement. When the piston is at maximum speed (i.e. steady state), the forces are compensating each other. Moreover, given that the density of the piston+car is much higher than the one of the oil, we can assume that the buoyancy is negligible in front of the weight. (Buoyancy is the Archimedes' Force which is equal to $\rho_{oil} g V_{piston}$)

Therefore:

$$\vec{F}_{gravity} + \vec{F}_{shear} = 0$$

The shear stress on the piston is given by:

$$\tau_{xy} = -\mu \frac{dv_y}{dx} = -\mu \frac{v_{max}}{Y}$$

And

$$F_{shear} = \tau_{xy} \times Area$$

$$Area = 2\pi R_1 h$$

So, we have:

$$-\mu\frac{v_{max}}{Y} \times 2\pi R_1 h + mg = 0$$

$$\boldsymbol{v_{max} = \frac{mgY}{2\pi R_1 h\mu}}$$

$$v_{max} = \frac{1.5 \times 10^3 \times 9.81 \times 2.5 \times 10^{-5}}{2\pi \times 0.1 \times 1 \times 0.707} = \mathbf{0.83\ m \cdot s^{-1}}$$

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